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LETTERE ALLA REDAZIONE

(La responsabilità scientifica degli scritti inseriti in questa rubrica è completamente lasciata dalla Direzione del periodico ai singoli autori)

Crystal Method for Measuring the Linear Polarization of Photons in the Multi-GeV Region.

G. BARBIELLINI, G. BOLOGNA, G. DIAMBRINI and G. P. MURTAS

Laboratori Nazionali di Frascati del C.N.E.N. - Frascati

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In a previous paper (1) we presented experimental data concerning the linear polarization of 150 MeV photons. We obtained these data by measuring the asymmetry in electron pair production at certain angles, in an «amorphous» converter. This method is not feasible when the photon energy is larger than $\approx$ 500 MeV, because the angle of emission of the pair particles, with respect to the primary photon, becomes too small.

In this letter we propose a method for measuring the linear polarization based again on asymmetry in pair production, but taking into account the pair particles emitted at *any angle* in a *thin crystal* converter (2).

The limitations in the use of this method are also examined.

Let us consider a high-energy photon beam of momentum $\boldsymbol{k}/c$ (c is the velocity of the light) linearly polarized in the direction $\boldsymbol{\epsilon}$ and striking a single crystal at a small angle θ ($\leqslant 0.1$ rad) with a reciprocal lattice axis $\boldsymbol{b}_1$. We choose two other axes $\boldsymbol{b}_2$, $\boldsymbol{b}_3$ perpendicular to each other and to $\boldsymbol{b}_1$, in order to have a reference frame. We started from MAY's (3) electron pair production differential cross-section from linearly polarized photons in the field of a nucleus and we used the same calculation technique employed in other works (1,4,5). We considered the electron pair production cross-section $\mathrm{d}\sigma_{\parallel}$ and $\mathrm{d}\sigma_{\perp}$ for $\boldsymbol{\epsilon}$ parallel and perpendicular, respectively, to the plane $(\boldsymbol{k}, \boldsymbol{b}_1)$, determined by the direction of the incident photons and the axis $\boldsymbol{b}_1$. $\mathrm{d}\sigma_{\parallel}$ and $\mathrm{d}\sigma_{\perp}$ are differential in the energy E_- of the electrons and integrated over the emission angles of the pair electrons.

(1) G. BARBIELLINI, G. BOLOGNA, G. DIAMBRINI and G. P. MURTAS: *Phys. Rev. Lett.*, **9**, 396 (1962).
(2) An alternative method based on photon absorption in a thick crystal converter was proposed by N. CABIBBO, G. DA PRATO, G. DE FRANCESCHI and U. MOSCO: *Phys. Rev. Lett.*, **9**, 270 (1962).
(3) M. MAY: *Phys. Rev.*, **84**, 265 (1951).
(4) G. BARBIELLINI, G. BOLOGNA, G. DIAMBRINI and G. P. MURTAS: *Phys. Rev. Lett.*, **8**, 454 (1962); *Erratum*, **9**, 46 (1962).
(5) H. ÜBERALL: *Zeits. Naturfor.*, **17**, 332 (1962).

As a result of the calculation we obtained the asimmetry ratio

$$R_c = \frac{\mathrm{d}\sigma_{\parallel} - \mathrm{d}\sigma_{\perp}}{\mathrm{d}\sigma_{\parallel} + \mathrm{d}\sigma_{\perp}}, \tag{1}$$

where $\mathrm{d}\sigma = \mathrm{d}\sigma_{\parallel} + \mathrm{d}\sigma_{\perp}$ represents the cross-section from unpolarized photons. We have

$$\left\{\begin{aligned} &\frac{\mathrm{d}\sigma_{\parallel} - \mathrm{d}\sigma_{\perp}}{N\bar{\sigma}\,\mathrm{d}y} = 2y(1-y)\,\psi_3(\theta, \delta)\,, \\ &\frac{\mathrm{d}\sigma_{\parallel} + \mathrm{d}\sigma_{\perp}}{N\bar{\sigma}\,\mathrm{d}y} = [y^2 + (1-y)^2][\psi_1(\theta, \delta) + \psi_1^c(\delta)] + \tfrac{2}{3}y(1-y)[\psi_2(\theta, \delta) + \psi_2^c(\delta)]\,, \end{aligned}\right. \tag{2}$$

where N is the number of atoms in the crystal, $y = E_-/k$, $\bar{\sigma} = Z^2(e^2/mc^2)^2/137$; $\delta = mc^2/2ky(1-y)$ is the minimum momentum transferred to the nucleus in units of mc (mc^2 is the rest mass of the electron). Z is the atomic number and e the electron charge. It results

$$\left\{\begin{aligned} &\psi_1(\theta, \delta) = 4\,\frac{N_0}{N}\,\frac{(2\pi)^2}{\Delta}\,\delta \sum_{\boldsymbol{g}} |S|^2\, \frac{\exp(-Ag^2)}{(\beta^{-2} + g^2)^2}\, \frac{g_2^2 + g_3^2}{[g_1 + (g_2\cos\alpha + g_3\sin\alpha)\,\theta]^2}\,, \\ &\psi_2(\theta, \delta) = \\ &\quad = 24\,\frac{N_0}{N}\,\frac{(2\pi)^2}{\Delta}\,\delta^2 \sum_{\boldsymbol{g}} |S|^2\, \frac{\exp(-Ag^2)}{(\beta^{-2} + g^2)^2}\, \frac{(g_2^2 + g_3^2)[g_1 + (g_2\cos\alpha + g_3\sin\alpha)\,\theta - \delta]}{[g_1 + (g_2\cos\alpha + g_3\sin\alpha)\,\theta]^4}\,, \\ &\psi_3(\theta, \delta) = -4\,\frac{N_0}{N}\,\frac{(2\pi)^2}{\Delta}\,\delta^3 \sum_{\boldsymbol{g}} |S|^2\, \frac{\exp(-Ag^2)}{(\beta^{-2} + g^2)^2}\, \frac{(g_2^2 - g_3^2)\cos 2\alpha + 2g_2 g_3 \sin 2\alpha}{[g_1 + (g_2\cos\alpha + g_3\sin\alpha)\,\theta]^4}\,, \end{aligned}\right. \tag{3}$$

In these formulas Δ is the volume of the fundamental cell, in units of $\lambda\!\!\!^{-}{}_C^3$ ($\lambda_C = 2\pi\lambda\!\!\!^{-}{}_C$, λ_C is the Compton wave length of the electron), N_0 is the number of fundamental cells in the crystal, S is the Bragg structure factor ([6]), and $\boldsymbol{g}$ is a reciprocal lattice vector; g_j ($j = 1, 2, 3$) are its components in the reference frame $\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3$, and have the form

$$g_j = \frac{2\pi}{a_j}\,h_j \qquad (h_j \text{ integer number});$$

α is the dyhedral angle between the planes $(\boldsymbol{b}_1, \boldsymbol{b}_2)$ and $(\boldsymbol{b}_1, \boldsymbol{k})$.

The other quantities in formulas (2) and (3) are defined and given by ÜBERALL ([7]).

The triple sum in formulas (3) is extended over the triplets $\boldsymbol{g} \equiv (g_1, g_2, g_3)$ satisfying the relationship

$$g_1 + (g_2\cos\alpha + g_3\sin\alpha)\,\theta \geqslant \delta\,.$$

([6]) In ref. ([4]) we called structure factor the quantity $(N_0/N)^{\frac{1}{2}}S$.

([7]) H. ÜBERALL: *Phys. Rev.*, **103**, 1055 (1956).

Note that S is defined with reference to the fundamental axes of the crystal, whilst g_j are referred to the axes $\boldsymbol{b}_j$, not necessarily coincident with the fundamental ones. The introduction of the axes $\boldsymbol{b}_j$ is convenient as the coherence effect takes place only at small angles θ between the photon direction and a lattice axis.

If we put $g_1 = 0$ we find again the functions given for bremsstrahlung by ÜBERALL ([5]) and by us in ref. ([1]) (for $\alpha = \pi/2$) and ref. ([4]) (for $\alpha = 0$).

Let us consider a diamond-like crystal. We have $N_0/N = \frac{1}{8}$ and $\Delta = a^3$, a being the edge of the fundamental cube in units of λ_C. The structure factor is given by

$$S = \{1 + \exp[i\pi(n_1 + n_2)] + \exp[i\pi(n_2 + n_3)] + \exp[i\pi(n_3 + n_1)]\} \cdot \cdot \left\{1 + \exp\left[i\frac{\pi}{2}(n_1 + n_2 + n_3)\right]\right\},$$

where (n_1, n_2, n_3) is any triplet of integer numbers.

Let us assume $\boldsymbol{b}_1 \equiv [110]$, $\boldsymbol{b}_2 \equiv [1\bar{1}0]$, $\boldsymbol{b}_3 \equiv [001]$; it follows $a_1 = a_2 = a/\sqrt{2}$, $a_3 = a$.

The structure of the reciprocal lattice planes $h_1 = 2l_1$, l_1 being any integer number, is given in Fig. 1*a* of ref. ([4]). The structure of the planes $h_1 = 2l_1 + 1$ is about the same, with the only difference that now the axis [110] intersects these planes in the

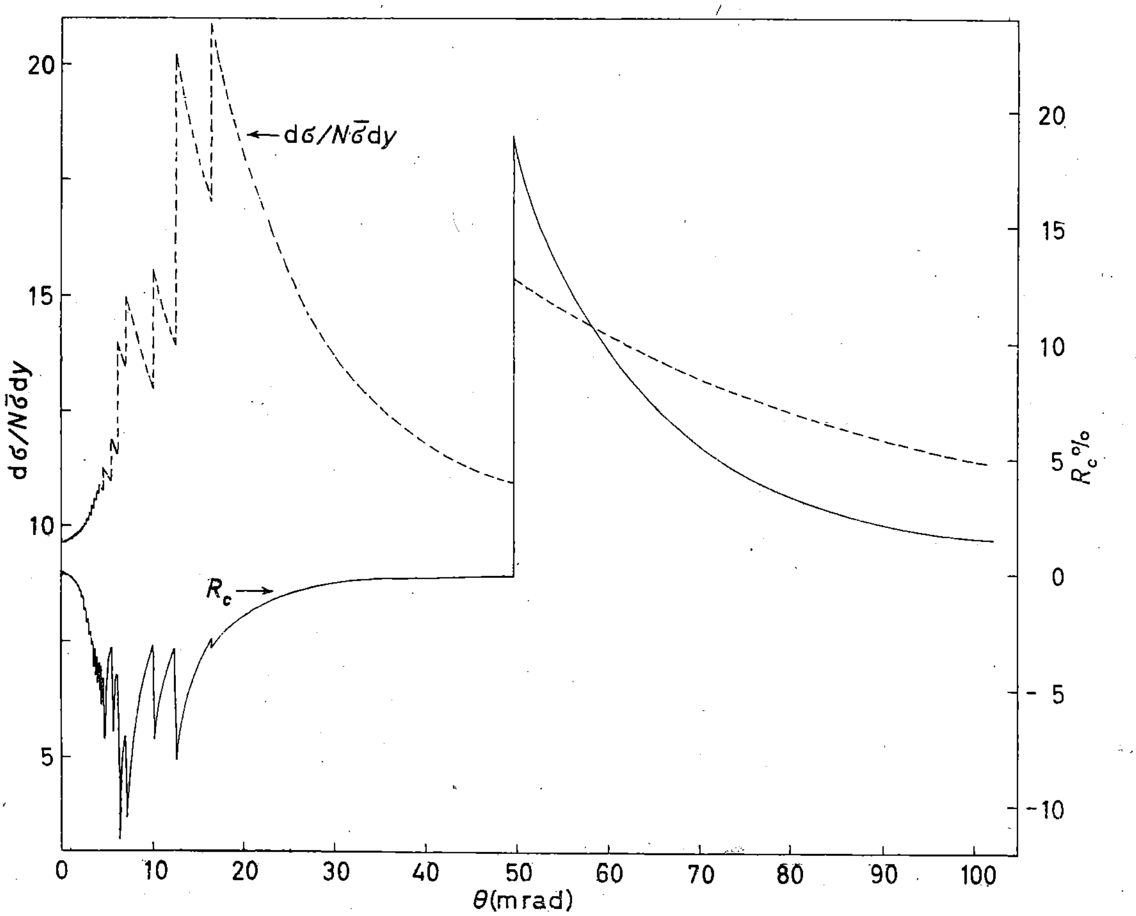

Fig. 1. – Electron pair production from $k = 3$ GeV linearly polarized photons in a diamond crystal at room temperature. The abscissa is the angle θ (in mrad) between the primary photon and the crystal axis [110]. The direction of the photons is parallel to the plane of the crystal axes [110] and [001]. The continuous curve represents in % the asymmetry ratio given by formula (1) (read at the right scale) and the dashed curve represents the cross-section from unpolarized photons (read at the left scale), given by the second of formulas (2), for symmetrical pairs.

centre of a rectangle, the vertices of which are in the nearest four points having $|S|^2 = 64$ (points enclosed in the circles in Fig. 1*a* of ref. (4)).

Numerical calculations were made for a diamond crystal at room temperature, assuming $A = 129$, $a/2\pi = 147$, $\beta = 61.1$, $\alpha = \pi/2$, with the choice of the preceding axes. These conditions are very convenient in order to obtain a large asymmetry ratio.

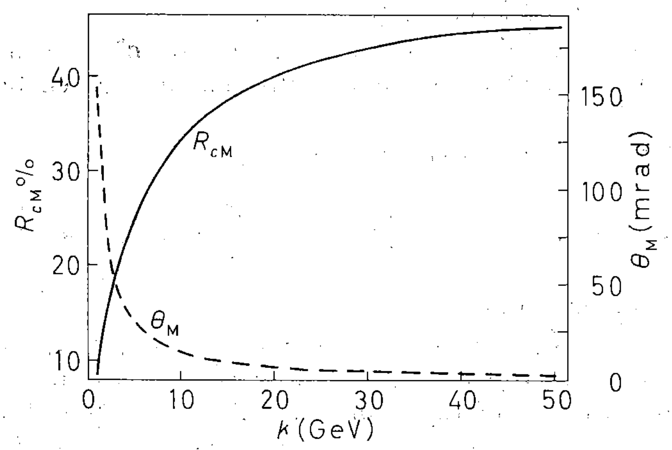

Fig. 2. – The abscissa is the energy k of the photons. The continuous curve gives the maximum value R_{cM} of the asymmetry ratio (the scale is at the left and is in %) and the dashed one gives the angle θ_M (the scale is at the right and is in mrad) for which R_{cM} is obtained. The other conditions are the same as in Fig. 1.

Some results are displayed in Fig. 1 as a function of the angle θ between the direction of the primary photon and the axis [110], for symmetrical pairs ($y = \frac{1}{2}$) and a photon energy $k = 3$ GeV. The continuous curve represents the asymmetry ratio R_c given by formula (1) (read at the right scale); the dashed curve represents the cross-section from unpolarized photons given by the second of formulas (2) (read at the left scale). One can see that the largest value of R_c is 19.0%, for $\theta = 49.5$ mrad.

Let us choose now at each energy k the best value of θ, *i.e.*, the value θ_M for which the largest value R_{cM} of R_c is obtained. R_{cM} and θ_M are represented as a function of k in Fig. 2. (The continuous curve and the left scale are for R_{cM}; the dashed curve and the right scale are for θ_M.)

The contribution of the planes $h_1 \neq 0$ is negligible down to $k \simeq 1$ GeV. For smaller energies this contribution becomes important, but the value of R_{cM} remains very small, as in the case of the plane $h_1 = 0$.

In order to measure the linear polarization of a photon beam (not necessarily monochromatic but with $k \geqslant 1$ GeV) one can use a simple apparatus, similar to that employed by us in an another work on coherent pair production (8). Let the directions of the photons be parallel and let the beam hit a thin (9) analiser diamond crystal at room temperature, contained in a pair spectrometer. Let us choose a particular energy k by fixing the spectrometer magnetic field and detect the asymmetrical electron pairs emitted at any angle by means of a conventional system of detectors. The number of symmetrical pairs detected depends on the cross-sections $d\sigma_{\parallel}$ and $d\sigma_{\perp}$ already calculated and on the amount and direction of the photon beam polarization. Let us arrange the analyser with the axis [110] at an angle θ with $\boldsymbol{k}$ slightly larger than θ_M and place the plane of the axes [110] and [001] parallel to $\boldsymbol{k}$. Then rotate the analyser around the direction of $\boldsymbol{k}$ until the largest counting rate is obtained. In this situation the polarization of the photon beam is parallel to the plane determined by $\boldsymbol{k}$ and [110]. Let $N_{\parallel}$ be the number of symmetrical pairs detected per fixed number of photons. Then rotate the analyser by 90° around $\boldsymbol{k}$; let $N_{\perp}$ be the correspondent numbers of pairs.

(8) G. BOLOGNA, G. DIAMBRINI and G. P. MURTAS: *Phys. Rev. Lett.*, **4**, 134 (1960).

(9) We consider the crystal thin when the mean square angle of scattering of the pair electrons is much less than the vertical angle of acceptance of the detectors.

The quantity which is meaningful for the experimental possibilities is the relative difference between $N_\parallel$ and $N_\perp$. It is given by

$$D = \frac{N_\parallel - N_\perp}{N_\perp} = \frac{2R_{cM}P}{1 - R_{cM}P}, \tag{4}$$

R_{cM} being given in Fig. 2 and P being the polarization of the photon beam, given by

$$P = \frac{n_\perp - n_\parallel}{n_\perp + n_\parallel};$$

$n_\perp$ and $n_\parallel$ are the number of photons in the beam with the polarization perpendicular and parallel, respectively, to the plane ($\boldsymbol{k}$, [110]).

The measurement offers no difficulty whenever $k \geqslant 1$ GeV; below this value R_{cM} is too small, so that, with the usual values of P, D becomes smaller than 5%, which is the lowest measurable value. From formula (4) we obtain

$$P = \frac{1}{R_{cM}} \frac{D}{2 + D}. \tag{5}$$

In order to obtain precise values of P it is necessary to measure D with a good statistical accuracy and to correct R_{cM} for the pair production in the field of the electrons, for the unavoidable angular divergence of the photon beam, and for the energy acceptance of the detectors. This is a straightforward matter in each particular experiment and we do not take it into account.

The error propagated on P by the statistical error of the counts is

$$\mu_P = \frac{2}{R_{cM}(2 + D)} \sqrt{\frac{1 + D}{(2 + D)\, N_\perp}}. \tag{6}$$

Owing to the large values of the pair production cross-sections, it seems reasonable to collect a number of counts $N_\perp \approx 2 \cdot 10^4$, provided that the photon beam intensity is not too small. Let us consider the favourable case $D = 50.0\%$. From formulas (5), (6) we obtain

$$\frac{\mu_P}{P} = 2.2\%.$$

As an unfavourable case we consider $D = 5.0\%$. We have

$$\frac{\mu_P}{P} = 20.0\%.$$

We notice that this type of measurement is very much simpler than the measurement performed in the previous work ([1]), in which we had to operate an angular selection of one particle in the pair.

In ref. ([1]) we gave the calculated polarization of $k = 150$ MeV bremsstrahlung photons from $E_1 = 1$ GeV electrons in a single diamond crystal.

We believe it is useful to present here some new numerical results for larger energies.

Let us consider a perfectly parallel collimated and monoenergetic electron beam whose energy is E_1. This beam hits a thin diamond crystal at room temperature at an angle θ_b with the crystal axis [110]; the direction of the electrons is parallel to the plane determined by the axes [110] and [001]. The entire bremsstrahlung γ-ray beam has a polarization which we calculated by means of the formulas given in ref. (1).

Among all the possible values of θ_b let us choose the angle θ_{bM} for which the polarization has its maximum value P_M at a given photon energy k. The results are given in Fig. 3 as a function of $x=k/E_1$ (the fractional energy of the photons) for three representative values of E_1 (1, 6, 40 GeV), corresponding to existing or future accelerators. In the figure are represented the values of P_M (read at the left scale) and θ_{bM} (read at the right scale).

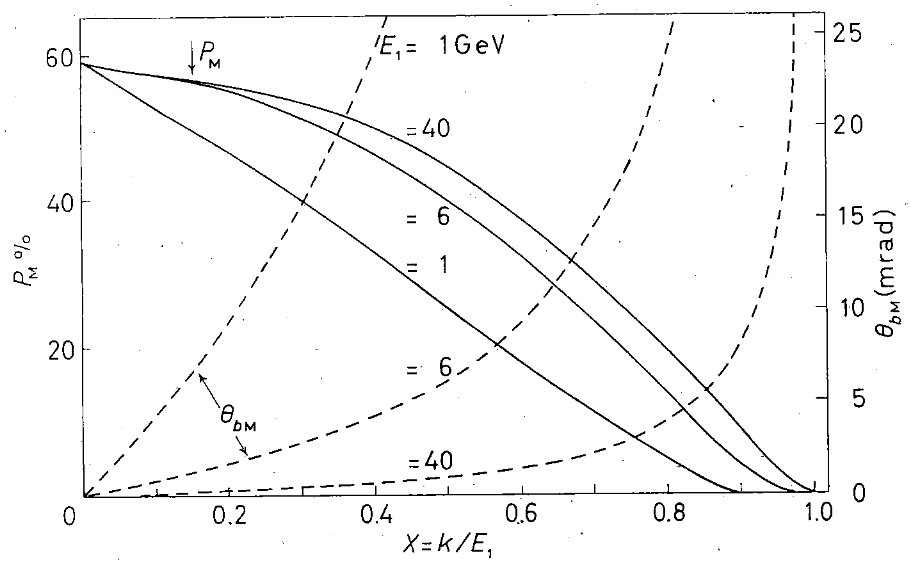

Fig. 3. – Polarization of the entire bremsstrahlung beam from electrons of three different energies E_1 in a thin diamond single crystal at room temperature. The direction of the electrons is parallel to the plane of the crystal axes [110], [001]. The abscissa is the fractional energy of the photons $x=k/E_1$. The continuous curves represent the maximum value P_M of the polarization (the scale is at the left and is in %), as computed by means of the formulas given in ref. (1), and the dashed curves represent the angle θ_{bM} for which P_M is obtained (the scale is at the right and is in mrad). θ_{bM} is the angle between the direction of the primary electrons and the crystal axis [110].

Experimental difficulties arise for $E_1 = 40$ GeV, if one is interested in photon energies $k \leqslant 20$ GeV. In this case $\theta_{bM} \leqslant 1$ mrad, a very small angle.

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